Values of the function & its derivatives and its relations in graphing.

$y = f(x)$

$f(x) =$ 0 when it hits the x-axis

$+$ above the x-axis

$-$ below the x-axis.

$f'(x) = 0$ or undefined when x is a **critical point**

(slope $= 0$ or undefined)

$f'(x) = +$ function is increasing

$f'(x) = -$ function is decreasing

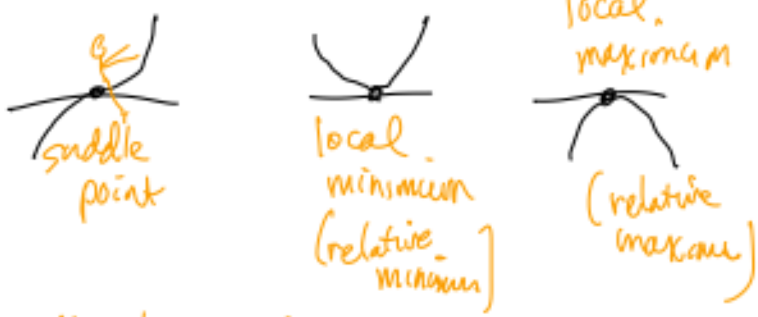
$f''(x) = 0$ might be an inflection point.

$f''(x) = +$ concave up (cc up)

$f''(x) = -$ concave down (cc down)

When you find critical points ($f'(x) = 0$ or undefined)

— there 3 different types:



Can tell what kind by looking at the slope:

eg $f'(x) = + + + + 0 - - - - - \Rightarrow$ local max.

Example Let $g(x) = (x^2 - 1)e^{-x}$. Find all critical points & inflection points, and determine the type of each critical point. After this, graph the function.

$g(x) = (x^2 - 1)e^{-x}$

when is this zero?

solution $g(x) = 0$

$(x+1)(x-1)e^{-x} = 0$

$x = -1$ or $+1$; e^{-x} is $+$

$$\begin{array}{l|ccccccccc} g(x) & +++ & 0 & - & - & - & 0 & +++ \\ x & & -1 & & 0 & & 1 & \end{array}$$

Find the critical points : $g'(x)=0$ or undefined.

$$g'(x) = 2xe^{-x} + \underbrace{(x^2-1)e^{-x}(-1)}_{(-x^2+1)e^{-x}} = (-x^2+2x+1)e^{-x} = 0$$

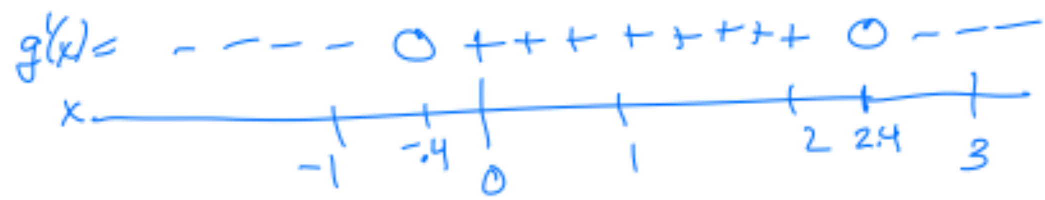
$$= -(x^2-2x-1)\,e^{-x} = 0$$

$\sqrt{8} = \sqrt{4\cdot 2} = \sqrt{4}\sqrt{2}$
$= 2\sqrt{2}$

↑ roots $x = \dfrac{2 \pm \sqrt{4+4}}{2} = \dfrac{2 \pm \sqrt{8}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2}$

$$= 1 \pm \sqrt{2}$$

$$\Rightarrow g'(x) = -\Big(x - \underbrace{(1-\sqrt{2})}_{\approx -.4}\Big)\Big(x - \underbrace{(1+\sqrt{2})}_{\approx 2.4}\Big)e^{-x}$$



$\Rightarrow g(x)$

$y = g(x)$ has critical points

$x = 1-\sqrt{2}$ (local min)

$x = 1+\sqrt{2}$ (local max).

2nd derivative $\quad g'(x) = -(x^2-2x-1)\,e^{-x} = (-x^2+2x+1)e^{-x}$

$$g''(x) = (-2x+2)e^{-x} + \underbrace{(-x^2+2x+1)e^{-x}(-1)}_{(x^2-2x-1)e^{-x}}$$

$$\Rightarrow g''(x) = (x^2-4x+1)e^{-x} = 0$$

/

$$\Rightarrow \ \hookrightarrow^{\text{roots}} \ x = \frac{4 \pm \sqrt{16-4}}{2}$$

$$x = \frac{4 \pm \sqrt{12}}{2} \qquad \sqrt{12} = \sqrt{4\cdot 3} = \sqrt{4}\,\sqrt{3} = 2\sqrt{3}$$

$$x = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3} \ \text{roots}$$

$$\Rightarrow \quad g''(x) = \Big(x - \underbrace{(2-\overset{1.7}{\sqrt{3}})}_{\approx .3}\Big)\Big(x - \underbrace{(2+\overset{1.7}{\sqrt{3}})}_{\approx 3.7}\Big)e^{-x}$$

	cc up		cc down		cc up
$g''(x) =$	$+ + + + +$	0	$- - - - - -$	0	$+ + +$
$x =$	$-1 \quad 0$	$.3$	$1 \quad 2 \quad 3$	3.7	4

$\therefore$ since the concavity changed at $x = 2-\sqrt{3}$ & $x = 2+\sqrt{3}$, those are the two inflection pts.

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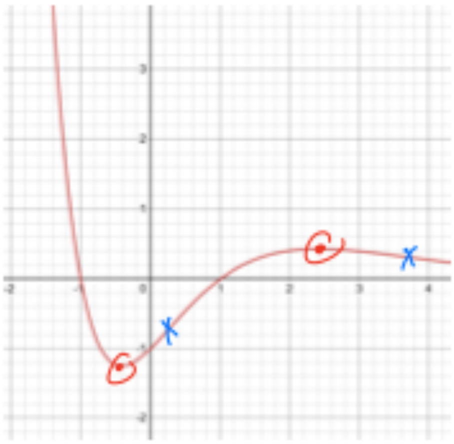
graph using sagemath: (see separate doc)

<u>Note:</u> inflection points occur exactly where the slope ($g'(x)$) has a local max or a local min.



$y = g(x)$

cr. pts

inflection pts.

If a function $f : [a,b] \to \mathbb{R}$ is defined on $[a,b]$, a global maximum (absolute maximum) is the x-value(s) for which the function is the greatest.

A global minimum is the x-value(s) for which the function is the least.

We can use the slope information from the derivative to help us find these, but we also need to look at the end behavior of the function — on the ends of the interval where it is defined.

Example: Find the global maximum and minimum of the function
$F(x) = \frac{1}{x} - \frac{2}{3-x}$ on the interval $(0, 1]$.

$$F'(x) = [x^{-1}]' - 2[(3-x)^{-1}]'$$
$$= -1x^{-2} - 2(-1)(3-x)^{-2}(-1)$$
$$= -x^{-2} - 2(3-x)^{-2}$$
$$\Rightarrow F'(x) = \frac{-1}{x^2} - \frac{2}{(3-x)^2}$$
$$F'(x) = 0 = \frac{-(3-x)^2 - 2x^2}{x^2(3-x)^2}$$

... . to be continued ... ..